

MOTION

LINEAR MOTION: This involves a body moving in a straight line under the action of forces.

Terms used

A. Speed: this is the rate of change of distance moved with time. It is expressed as $speed = \frac{\text{distance moved}}{\text{time taken}}$.

Its S.I units are *metres per second* (ms^{-1}) other units are kmh^{-1}

$$1km = 1000m \text{ and } 1h = 3600s \Rightarrow \frac{1km}{1h} = \frac{1000m}{3600s} = \frac{5}{18}ms^{-1}$$

$$\text{Thus } 1kmh^{-1} = \frac{5}{18}ms^{-1} \text{ or } 1ms^{-1} = \frac{18}{5}kmh^{-1}$$

Example

1. A cat moved from point A to B 20m apart in 1hour. What was its speed?

Solution

$$\text{Time } t = 1\text{hour} = 60 \times 60 = 3600s$$

$$speed = \frac{\text{distance moved}}{\text{time taken}} = \frac{20}{3600} = 0.0056ms^{-1}$$

2. Change $50kmh^{-1}$, $72kmh^{-1}$ in ms^{-1} and $45ms^{-1}$ in kmh^{-1}

Solution:

$$50kmh^{-1} = \frac{5}{18} \times 50 = 13.89ms^{-1}, 72kmh^{-1} = \frac{5}{18} \times 72 = 20ms^{-1},$$

$$45ms^{-1} = \frac{18}{5} \times 45 = 162kmh^{-1}$$

- B. Displacement:** this is the distance moved in a specified direction. Its units are metres (m).
- C. Velocity:** this is the rate of change of displacement with time. Its S.I units are ms^{-1} other units are kmh^{-1}
- D. Constant / uniform velocity:** if a body moves equal distances in equal times it is said to be moving with a uniform velocity.

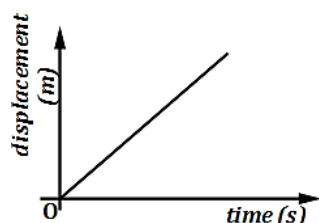
Example: an athlete covered a certain distance in a given time as in the table below

Distance (m)	0	5	10	15	20	25	60
Time (s)	0	1	2	3	4	5	6

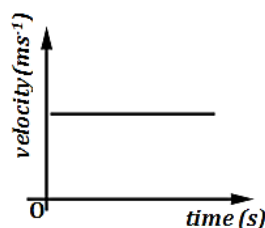
From the table, for every 5m the athlete takes 1 second. Therefore the uniform speed of the athlete is $\frac{5m}{1s} = 5ms^{-1}$

Graphs of uniform velocity

Distance – time graph

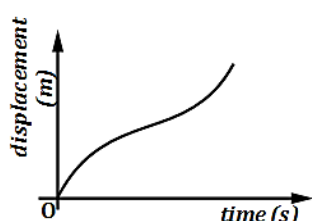


Velocity – time graph

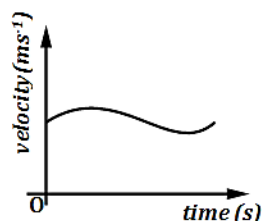
Graphs of non-uniform velocity

Distance covered by a body may not increase by the same amounts in the same time. This makes its velocity to keep changing.

Distance – time graph



Velocity – time graph



E. Acceleration: this is the rate of change of velocity with time.

acceleration, $a = \frac{\text{velocity change}}{\text{time taken}} = \frac{v-u}{t}$. Where u is the initial velocity (when timing started), v is the final velocity (when timing stopped) and t is the time taken. Its S.I units are *metres per square second* (ms^{-2}).

Note: If $v > u$ a body's velocity is increasing, the body is said to be accelerating, otherwise it is decelerating / retarding.

Examples:

1. A saloon car started from rest and accelerated to 48kmh^{-1} in 10seconds. Find its acceleration.

Solution: $48\text{kmh}^{-1} = \frac{5}{18} \times 48 = 13.3\text{ms}^{-1}$, at rest, $t = 0\text{s}$ and $u = 0\text{ms}^{-1}$,

after $t = 10\text{s}$, $v = 13.3\text{ms}^{-1}$

$$a = \frac{v-u}{t} = \frac{13.3-0}{10} = 1.33\text{ms}^{-2}$$

2. The table below shows the velocity of a car at different times

velocity (kmh^{-1})	20	56	40
Time (s)	10	20	25

Calculate the acceleration of the car between

- i. 10th and 20th second ii. 20th and 25th second

Solution:

$$20\text{kmh}^{-1} = \frac{5}{18} \times 20 = 5.56\text{ms}^{-1}, \quad 56\text{kmh}^{-1} = \frac{5}{18} \times 56 = 15.56\text{ms}^{-1} \quad \text{and}$$

$$40\text{kmh}^{-1} = \frac{5}{18} \times 40 = 11.11\text{ms}^{-1}$$

$$(i) \quad u = 5.56\text{ms}^{-1}, \text{ after } t = 20 - 10 = 10\text{s}, \quad v = 15.56\text{ms}^{-1}$$

$$a = \frac{v-u}{t} = \frac{15.56-5.56}{10} = 1\text{ms}^{-2}$$

$$(ii) \quad u = 15.56\text{ms}^{-1} \text{ after } t = 25 - 20 = 5\text{s}, \quad v = 11.11\text{ms}^{-1}$$

$$a = \frac{v-u}{t} = \frac{11.11-15.56}{5} = -0.89\text{ms}^{-2} \quad \text{The negative means the car was decelerating at a rate of } 0.89\text{ms}^{-2}$$

Questions

1. A train travels at 36kmh^{-1} and accelerates to 108kmh^{-1} in 10s, calculate its acceleration in ms^{-2} .

2. k

- F. Uniform acceleration:** a body moves with a uniform acceleration if its velocity increases by equal amounts in equal times.

Example:

1. The table below shows the velocity of a car in a given time.

velocity (kmh^{-1})	0	10	20	30	40
Time (s)	0	5	10	15	20

From the table, for every $10\text{kmh}^{-1} = 10 \times \frac{5}{18} = 2.78\text{ms}^{-1}$ the car takes 5s.

Therefore the uniform acceleration of the car is $\frac{2.78\text{ms}^{-1}}{5\text{s}} = 0.556\text{ms}^{-2}$

2. A car is moving at 108kmh^{-1} is uniformly brought to rest in 15s. Find its acceleration.

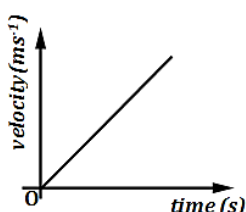
$$108\text{kmh}^{-1} = 108 \times \frac{5}{18} = 30\text{ms}^{-1}$$

$$u = 30\text{ms}^{-1}, v = 0\text{ms}^{-1}, t = 15\text{s} \Rightarrow a = \frac{0-30}{15} = -2\text{ms}^{-2}$$

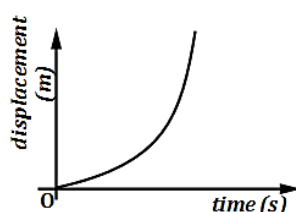
Its acceleration is -2ms^{-2}

Graphs of uniform acceleration

Velocity – time graph

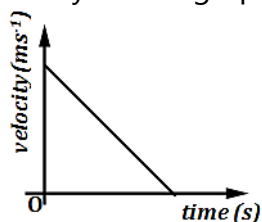


displacement – time graph

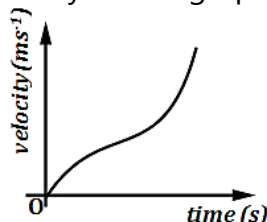


Graph of uniform deceleration

Velocity – time graph

Graph of non – uniform deceleration

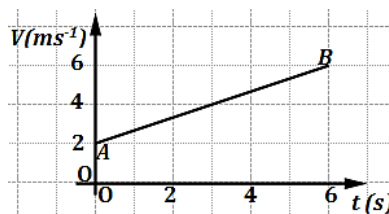
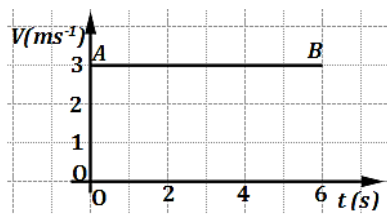
Velocity – time graph

**Calculation of distance from a velocity – time graph**

The area under a velocity – time (V – t) graph gives the distance travelled.

Examples

1. From the figures below, find the distance travelled by the cyclist from point A to point B.

Solution:

From graph 1, distance covered = $6 \times 3 = 18\text{m}$

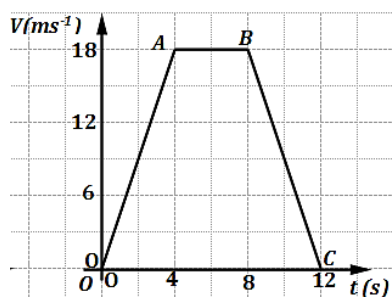
Distance travelled = 18m.

From graph 2, distance covered = $\frac{1}{2} \times 6(2 + 6) = 24\text{m}$

Or distance covered = $6 \times 2 + \frac{1}{2} \times 6 \times 4 = 24\text{m}$

Distance travelled = 24m.

2. A motorist moved from one center to the other.



(i) When is the motorist moving with constant speed?

(ii) Calculate the retardation of the motorist

(iii) Calculate the distance moved by the motorist

(iv) Calculate the distance moved by the motorist

in the 6th second.

(v) Describe the motion of the motorist.

Solution:

(i) Between AB the motorist moved with constant speed of 18ms^{-1} for 4 seconds.

(ii) Retardation is along BC; $u = 18\text{ms}^{-1}$, $v = 0\text{ms}^{-1}$, $t = 12 - 8 = 4\text{s}$

$$a = \frac{v-u}{t} = \frac{0-18}{4} = -4.5\text{ms}^{-2}$$

Retardation is 4.5ms^{-2}

(iii) Distance covered $= \frac{1}{2} \times 18(4 + 12) = 144\text{m}$ or

$$= \frac{1}{2} \times 4 \times 18 + 4 \times 18 + \frac{1}{2} \times 4 \times 18 = 144\text{m}$$

(iv) Distance covered $= \frac{1}{2} \times 18(2 + 6) = 72\text{m}$ or

$$= \frac{1}{2} \times 4 \times 18 + 2 \times 18 = 72\text{m}$$

(v) Between OA; $u = 0\text{ms}^{-1}$, $v = 18\text{ms}^{-1}$, $t = 4\text{s}$ $a = \frac{v-u}{t} = \frac{18-0}{4} = 4.5\text{ms}^{-2}$

The motorist moved from rest with a uniform acceleration of 4.5ms^{-2} for 4s

Between AB the motorist moved with constant speed of 18ms^{-1} for 4s.

Between BC the motorist moved to rest with a uniform deceleration of 4.5ms^{-2} for 4s

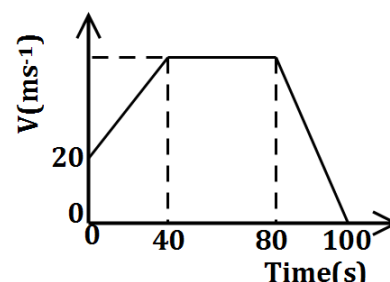
Questions

- A car moves from rest with a uniform acceleration of 1ms^{-2} for the first 20s. It continues at a constant velocity for the next 30s and finally takes 10s to decelerate uniformly to rest.
 - Calculate the constant speed reached after 20s.
 - Sketch a velocity – time graph for the whole journey.
 - Calculate the distance travelled by the car.
- A car travelling with a uniform velocity of 25ms^{-1} for 5s brakes and then comes to rest under a uniform deceleration in 8s.
 - Sketch a velocity – time graph for the motion
 - Find the deceleration
 - Find the total distance travelled.
- A lift accelerates uniformly from rest for 3s. It then moves at a uniform velocity of 40ms^{-1} for 15s then decelerates uniformly for 2s before coming to rest.

By sketching a velocity – time graph, find the total distance travelled by the lift.

4. The figure shows the motion of a car with an acceleration of 2 ms^{-2} .

- Describe the motion of the car
- Find the distance moved after 50s
- Find the total distance travelled by the car.



Equations of motion

If a body moves with initial velocity, u accelerates uniformly to attain a final velocity v and covers a distance, S in time t its acceleration $a \text{ ms}^{-2}$ is got from

$$a = \frac{v-u}{t} \Rightarrow v = u + at \dots \dots (1)$$

$$\text{Average speed} = \frac{\text{total distance}}{\text{time taken}}$$

$$\Rightarrow \frac{v+u}{2} = \frac{S}{t} \Rightarrow S = \left(\frac{v+u}{2}\right)t$$

Since $v = u + at$

$$\Rightarrow S = \left(\frac{u+at+u}{2}\right)t = \left(\frac{2u+at}{2}\right)t = \left(u + \frac{at}{2}\right)t = ut + \frac{1}{2}at^2$$

$$\Rightarrow S = ut + \frac{1}{2}at^2 \dots \dots (2)$$

$$\text{From } v = u + at \Rightarrow v^2 = (u + at)^2 = u^2 + 2uat + a^2t^2 = u^2 + 2a\left[ut + \frac{1}{2}at^2\right]$$

$$\text{Since } v = u + at \Rightarrow v^2 = u^2 + 2aS \dots \dots (3)$$

Calculations

- A body started from rest and accelerated uniformly at 2 ms^{-2} until it attains a speed of 36 ms^{-1} . Find the distance the body covered and the time taken.

$$\text{Solution: } u = 0 \text{ ms}^{-1}, v = 36 \text{ ms}^{-1}, a = 2 \text{ ms}^{-2}, s = ?, t = ?$$

$$\text{Using } v = u + at, \Rightarrow 36 = 0 + 2t \Rightarrow t = \frac{36}{2} = 18 \text{ s}$$

$$\text{Using } s = ut + \frac{1}{2}at^2, \Rightarrow s = 0 + \frac{1}{2} \times 2 \times 18^2 = 324 \text{ m}$$

- A car moving with uniform acceleration of 4 ms^{-2} increases its speed from 20 ms^{-1} to 60 ms^{-1} . Calculate the total time taken and the distance moved.

$$\text{Solution: } u = 20 \text{ ms}^{-1}, v = 60 \text{ ms}^{-1}, a = 4 \text{ ms}^{-2}, s = ?, t = ?$$

$$\text{Using } v = u + at, \Rightarrow 60 = 20 + 4t \Rightarrow 40 = 4t \Rightarrow t = 10 \text{ s}$$

$$\text{Using } s = ut + \frac{1}{2}at^2, \Rightarrow s = 20 \times 10 + \frac{1}{2} \times 4 \times 10^2 = 400 \text{ m}$$

Motion under gravity

Every object moving under the influence of gravity has an acceleration called *acceleration due to gravity* $g = 9.8 \approx 10\text{ms}^{-2}$. $g = +10\text{ms}^{-2}$ When a body is falling freely and $g = -10\text{ms}^{-2}$ when a body is thrown upward.

Acceleration due to gravity is defined as the rate of change of velocity of an object falling freely.

Example:

1. An orange fell from a tree 20m high. Find its speed at the time it hit the ground.

Solution

$$u = 0\text{ms}^{-1}, a = 10\text{ms}^{-2}, s = 20\text{m}, v = ?$$

$$\text{Using } v^2 = u^2 + 2aS \Rightarrow v^2 = 0^2 + 2 \times 10 \times 20 = 400 \Rightarrow v = 20$$

The stone hit the ground with a speed of 20ms^{-1}

2. A bomb dropped from an air craft flying at a height of 10,000m. With what velocity did it hit the ground?

Solution

$$u = 0\text{ms}^{-1}, a = 10\text{ms}^{-2}, s = 10,000\text{m}, v = ?$$

$$\text{Using } v^2 = u^2 + 2aS \Rightarrow v^2 = 0^2 + 2 \times 10,000 \times 20 = 400,000 \Rightarrow v = 632.5$$

The bomb hit the ground with a speed of 632.5ms^{-1}

3. A ball is thrown from the ground to the top of flag pole. If the ball returns to the ground in 6s, find the height of the flag pole.

Solution: let time from the ground to top of flag be t . At the turning point of the ball, it is momentarily at rest ($v = 0\text{ms}^{-1}$).

$$\text{Time from ground back to ground is } 2t \Rightarrow 2t = 6 \Rightarrow t = 3$$

$$u = ? \quad a = -10\text{ms}^{-2}, s = ? \text{m}, v = 0\text{ms}^{-1}, t = 3\text{s}$$

$$\text{Using } v = u + at \Rightarrow 0 = u - 10 \times 3 \Rightarrow u = 30\text{ms}^{-1}$$

$$\text{Using } S = ut + \frac{1}{2}at^2 \Rightarrow S = 30 \times 3 - \frac{1}{2} \times 10 \times 3^2 = 45\text{m}$$

The height of flag pole is 45m

4. A stone is thrown vertically upwards with a velocity of 14ms^{-1} . Neglecting air resistance find the
 - i. maximum height reached
 - ii. time taken before reaching the ground

Solution:

To reach maximum point; $u = 14\text{ms}^{-1}$, $a = -10\text{ms}^{-2}$, $s = ?\text{m}$, $v = 0\text{ms}^{-1}$, $t = ?$

(i) Using $v^2 = u^2 + 2aS \Rightarrow 0 = 14^2 - 2 \times 10S \Rightarrow S = 9.8\text{m}$

(ii) Using $v = u + at \Rightarrow 0 = 14 - 10t \Rightarrow t = 1.4\text{s}$

Time taken to reach the ground $= 2 \times 1.4 = 2.8\text{s}$

5. A bullet is fired vertically from a gun held 2m above the ground reaches a maximum height in 4seconds. Calculate
- its initial velocity
 - total distance travelled by the bullet to hit the ground.

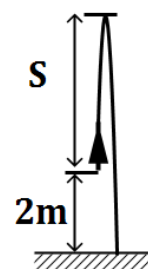
Solution:

To reach maximum point; $u = ?\text{ms}^{-1}$, $a = -10\text{ms}^{-2}$, $s = ?\text{m}$, $t = 4\text{s}$

(i) Using $v = u + at \Rightarrow 0 = u - 10 \times 4 \Rightarrow u = 40\text{ms}^{-1}$

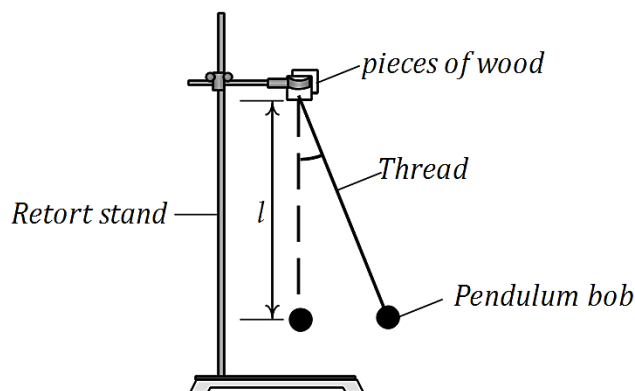
(ii) Using $v^2 = u^2 + 2aS \Rightarrow 0 = 40^2 - 2 \times 10S \Rightarrow S = 80\text{m}$

Total distance travelled by bullet to hit the ground $= 80 + 80 + 2 = 162\text{s}$



Experiment to determine acceleration due to gravity

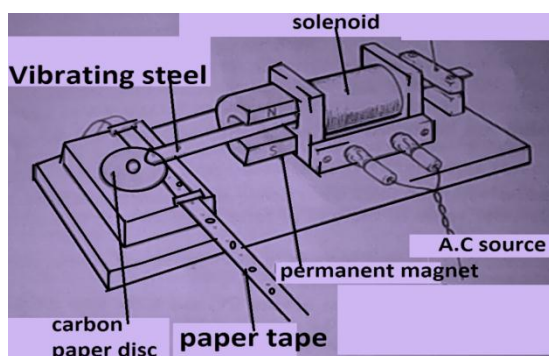
- a) the pendulum is suspended from a retort stand starting with a length of the thread, $l = 20.0\text{ cm}$ as in the figure below



- The pendulum bob is displaced through a small angle to the vertical and then released.
- The time for 20 oscillations is measured and recorded.
- Time, T , for one oscillation is obtained from $T = \frac{t}{20}$.
- The acceleration due to gravity, g , is obtained from $g = \frac{4\pi^2}{T^2}$

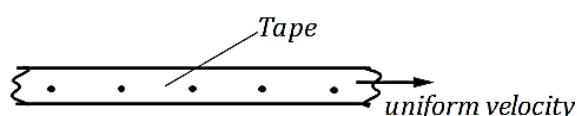
The tick-tape timer

A ticker-timer is an electric device which consists of a rapidly vibrating hammer that prints dots onto a length of paper pulled through it. The dots formed are used in studying motion of a trolley attached on the paper.



If distance between two consecutive dots is

- constant then the trolley moves with uniform velocity



- Increasing then the trolley is accelerating
- Decreasing then the trolley is decelerating.



Note: If the hammer makes 50 dots in one second, its frequency is 50Hz.

1 dot is made in $T = \frac{1}{50} = 0.02s$

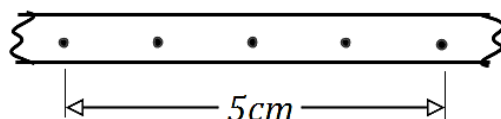
Thus the time interval between two consecutive dots is 0.02s. This time is called a *tick*.

E.g.

- The time taken to cover 9 intervals (10 dots) = $9 \times 0.02 = 0.18s$
- The time taken to cover 5 intervals (6 dots) = $5 \times 0.02 = 0.10s$
- The time taken to cover 12 intervals (13 dots) = $12 \times 0.02 = 0.24s$

Calculations

- Find the speed of the tape as in the figure below if the tick-timer operates at a frequency of 50Hz.

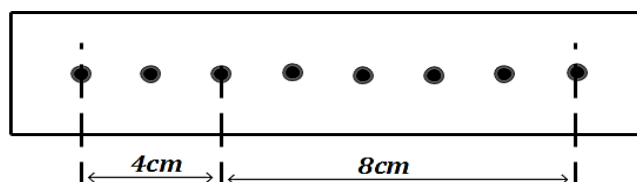


$$\text{Solution: Tick} = \frac{1}{50} = 0.02s$$

$$\text{Time taken to cover 5cm (4 intervals)} = 4 \times 0.02 = 0.08s$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{5}{0.08} = 62.5 \text{ cm s}^{-1}$$

- The figure shows a ticker tape pulled by a trolley through a ticker-timer. Describe the motion of a trolley, if the frequency is 50Hz.



Solution

$$\text{Solution: Tick} = \frac{1}{50} = 0.02s$$

$$\text{Time taken to cover 4cm (2 intervals)} = 2 \times 0.02 = 0.04s$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{0.04}{0.04} = 1ms^{-1}$$

$$\text{Time taken to cover 8cm (5 intervals)} = 5 \times 0.02 = 0.1s$$

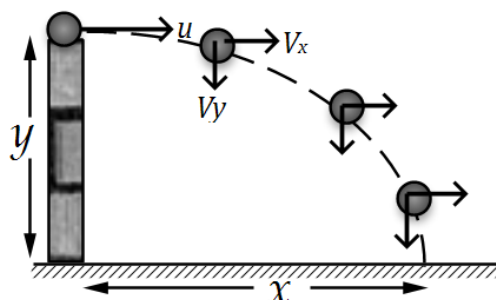
$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{0.08}{0.1} = 0.8ms^{-1}$$

If the trolley moves from left to right, it is accelerating since its speed is increasing.

$$a = \frac{1-0.8}{(0.04+0.1)} = 1.4286ms^{-2} \text{ Or total time taken} = 7 \times 0.02 = 0.14s$$

THE PROJECTILE MOTION

A projectile is an object which when given an initial velocity moves under the influence of gravity and is acted on by only its weight. The path taken by a projectile is called *trajectory*.



u is the initial velocity, V_x is horizontal velocity after time t and V_y is vertical velocity after time t

When the ball hits the ground, it will cover

- a vertical distance $s = y$ with acceleration $a = g = 10ms^{-2}$
- a horizontal distance $s = x$ with acceleration $a = 0ms^{-2}$.

Example

1. A ball is thrown horizontally with a speed of $15ms^{-1}$ from a top a building. If it takes 2 seconds to reach the ground, find the height of the building.

Solution:

For vertical motion; $u = 0ms^{-1}$ [since the ball was thrown horizontally],

$$a = -10ms^{-2}, t = 2s, S = ?$$

$$\text{Using } s = ut + \frac{1}{2}at^2, \Rightarrow s = 0 \times 2 + \frac{1}{2} \times 10 \times 2^2 = 20m$$

2. A stone is thrown horizontally with a velocity of 6ms^{-1} from the edge of the cliff 125m tall; find how far the stone landed from the bottom of the cliff.

Solution:

For vertical motion; $u = 0\text{ms}^{-1}$, $a = 10\text{ms}^{-2}$, $t = ?$, $S = 125\text{m}$

$$\text{Using } s = ut + \frac{1}{2}at^2 \Rightarrow 125 = 0 \times t + \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow t^2 = 25 \Rightarrow t = 5\text{s}$$

For horizontal motion; $u = 6\text{ms}^{-1}$, $a = 0\text{ms}^{-2}$, $t = 5\text{s}$, $S = ?$

$$\text{Using } s = ut + \frac{1}{2}at^2 \Rightarrow S = 6 \times 5 + \frac{1}{2} \times 0 \times 5^2 = 30$$

The ball landed 30m from the bottom of the cliff.

3. A bomb is released from a plane 5000m high with a velocity of 30ms^{-1} . Find the
- time it takes to reach the ground
 - horizontal distance it covers by the time it hits the ground

Solution:

For vertical motion; $u = 0\text{ms}^{-1}$, $a = 10\text{ms}^{-2}$, $t = ?$, $S = 5000\text{m}$

$$\text{Using } s = ut + \frac{1}{2}at^2 \Rightarrow 5000 = 0 \times t + \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow t^2 = 1000 \Rightarrow t = 31.6\text{s}$$

For horizontal motion; $u = 30\text{ms}^{-1}$, $a = 0\text{ms}^{-2}$, $t = 31.6\text{s}$, $S = ?$

$$\text{Using } s = ut + \frac{1}{2}at^2 \Rightarrow S = 30 \times 31.6 + \frac{1}{2} \times 0 \times 5^2 = 948\text{m}$$

The bomb hits 948m from the time it is released.

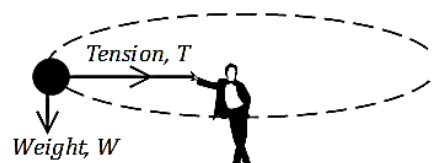
CIRCULAR MOTION

This is the motion of a body in a circle about a fixed point. The forces acting on the body are

- its weight (acting downwards)
- centripetal force (acting towards the centre of motion). This keeps the body in a circular path.

Circular motion is experienced by

- a whirled stone tied on string and centripetal force is the tension in the string.
- a car or bicycle rider negotiating a corner and centripetal force is the frictional force between the tyre and the ground.



- an aircraft making a circular turn and centripetal force is the frictional force between the wing and the air molecules.
- centrifuge which separates liquids of different densities

NEWTON'S LAWS OF MOTION

First law: Every object continues in its state of rest or uniform motion in a straight line unless acted upon by an external force.

Example: A rolling ball would keep rolling; a desk would remain in class room.

Inertia:

It is the tendency of a body to remain at rest or to keep moving with a uniform velocity.

Examples:

- A passenger jerks forwards when a bus is suddenly stopped or jerks backwards when a bus is suddenly started.
- Books placed on the cloth remain on table when the cloth is pulled faster.

Momentum

It is the product of mass of a body and its velocity. $momentum = m \times v$.

Its S.I units are $kgms^{-1}$ or Ns .

Example:

1. A bullet of mass 400g is fired from a gun with a velocity of $40ms^{-1}$. What is the momentum of the bullet?

$$momentum = mv = \frac{400}{1000} \times 40 = 16Ns$$

2. A body of mass 20kg is dropped and falls to the ground with a momentum of 800Ns. With what velocity did it have?

$$momentum = mv \Rightarrow 800 = 20v \Rightarrow v = \frac{800}{20} = 40ms^{-1}$$

Second law: The rate of change of momentum of a body is directly proportional to the applied force and it takes place in the direction of force.

Suppose a force F acts on an object of mass m to cause a velocity change from u to v , in time t ;

Change in momentum = $mv - mu$ and

Rate of change of momentum = $\frac{mv - mu}{t}$

$$\text{Thus } F \propto \frac{mv - mu}{t} = m \frac{(v - u)}{t} = ma$$

$$\therefore F = ma$$

A *newton* is a force required to give a body of mass 1kg an acceleration of 1ms^{-2} .

Example:

1. A bullet of mass 20g is uniformly retarded at from 320ms^{-1} and brought to rest in 0.05seconds. Calculate the retarding force.

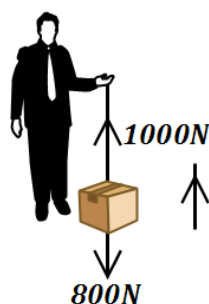
$$a = \frac{0 - 320}{0.05} = -6400\text{ms}^{-2}, \text{ using } F = ma \Rightarrow F = \frac{20}{1000} \times -6400 = -128\text{N}$$

$$\text{Retarding force} = 128\text{N}$$

2. A car of mass 200kg is acted on by a force of 4000N, find its acceleration.

$$\text{Using } F = ma \Rightarrow 4000 = 200a \Rightarrow a = \frac{4000}{200} = 20\text{ms}^{-2}$$

3. A box of mass 80kg is raised vertically using a rope. If the tension in the rope is 1000N find the acceleration of the box.

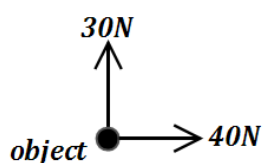


$$\text{Weight of the box } mg = 80 \times 10 = 800\text{N}$$

$$\text{Resultant force } 1000 - 800 = 200\text{N}$$

$$\text{Using } F = ma \Rightarrow 200 = 80a \Rightarrow a = \frac{200}{80} = 2.5\text{ms}^{-2}$$

4. Two forces of 30N and 40N act perpendicularly on an object of mass 10kg as shown in the figure below. Calculate the magnitude of acceleration of the object.



$$\text{Resultant force} = \sqrt{30^2 + 40^2} = \sqrt{2500} = 50\text{N}$$

$$\text{Using } F = ma \Rightarrow 50 = 10a \Rightarrow a = \frac{50}{10} = 5\text{ms}^{-2}$$

$$\text{Acceleration is } 5\text{ms}^{-2}$$

Conservation of linear momentum

It states that when two or more bodies collide, their total momentum remains constant provided no external force acts on them.

In calculations we use total momentum before collision is equal to total collision after collision $m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$

Collisions can either be elastic or inelastic

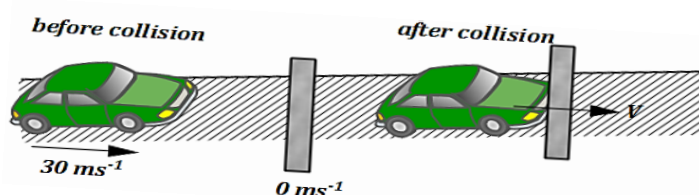
Elastic collision occurs when two bodies collide and move separately with different velocities after collision.

Inelastic collision occurs when two bodies collide and move with common/same velocity after collision.

Examples:

1. A car of mass 6000kg moving with a velocity of 30ms^{-1} collides and couples with a stationary truck of 3000kg. Find the velocity with which they move.

Let velocity of the car and truck after collision be v

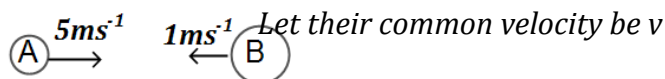


Momentum before collision = momentum after collision

$$6000 \times 30 = (6000 + 3000)v \Rightarrow v = \frac{180000}{9000} = 20\text{ms}^{-1}$$

They move with a velocity of 20ms^{-1} after collision

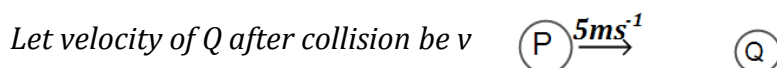
2. A ball A of mass 100g moving with a velocity of 5ms^{-1} collides head-on with ball B of mass 200g moving with a velocity of 1ms^{-1} . Find their common velocity after collision.



$$\begin{aligned} \text{Taking } \begin{array}{c} \rightarrow \\ + \end{array} \quad \begin{array}{c} \leftarrow \\ - \end{array} &\Rightarrow 100 \times 5 + 200 \times -1 = (100 + 200)v \\ &\Rightarrow 500 - 200 = 300v \Rightarrow 300 = 300v \Rightarrow v = 1\text{ms}^{-1} \end{aligned}$$

They move with a velocity of 1ms^{-1} in the direction of A

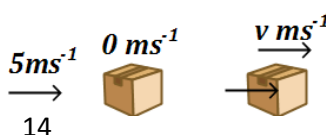
3. A ball P of mass 200g moving with a velocity of 5ms^{-1} collides with a stationary ball Q of mass 150g. If P moves backward with a velocity of 2ms^{-1} , find the velocity of Q after collision.



$$\begin{aligned} \text{Taking } \begin{array}{c} \rightarrow \\ + \end{array} \quad \begin{array}{c} \leftarrow \\ - \end{array} &\Rightarrow 200 \times 5 + 150 \times 0 = 200 \times -2 + 150v \\ &\Rightarrow 1000 = -400 + 150v \Rightarrow 1400 = 150v \Rightarrow v = 9.33\text{ms}^{-1} \end{aligned}$$

Q moves with a velocity of 9.33ms^{-1} in the direction of P

4. An arrow of mass 100g travelling at 5ms^{-1} is shot in stationary wooden block of mass 25kg placed on a smooth surface. Calculate the common velocity after impact.



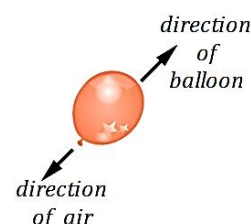
Let their common velocity be v

Taking $\overrightarrow{+}$ $\overleftarrow{-} \Rightarrow 100 \times 5 + 50 \times 0 = (100 + 25)v$
 $\Rightarrow 500 = 125v \Rightarrow v = 4\text{ms}^{-1}$

Q moves with a velocity of 4ms^{-1} in the direction of the bullet.

Applications of momentum

1. A rocket acquires an equal and opposite momentum as the exhaust gases. This makes it move upwards as the gases move backwards.
2. A filled balloon when released moves in an opposite direction as that of escaping air.
3. A gun is held tightly when bullets are fired. The gun acquires an equal and opposite momentum as that of the bullets



Example:

1. A bullet of mass 150g is fired with a speed of 400ms^{-1} . The rifle recoils at a speed of 10ms^{-1} . Find the mass of the rifle.

Momentum of the rifle = momentum of the bullet

$$\Rightarrow m \times 10 = 150 \times 400 \Rightarrow 10m = 60000 \Rightarrow m = 6000\text{g}$$

Mass of the rifle is 6000g

2. A bullet of mass 15g is fired from a gun of mass 3kg with a muzzle velocity of 42ms^{-1} . What is the velocity of the gun?

Momentum of the gun = momentum of the bullet

$$\Rightarrow 3 \times v = \frac{15}{1000} \times 42 \Rightarrow 3v = 0.63 \Rightarrow v = 0.21\text{ms}^{-1}$$

Velocity of the gun is 0.21ms^{-1}

3. A bullet of mass 0.1kg is fired from a rifle of mass 5kg . The rifle recoils with a velocity of 16ms^{-1} . Calculate the velocity with which the bullet is fired.

Momentum of the gun = momentum of the bullet

$$\Rightarrow 5 \times 16 = 0.1 \times v \Rightarrow v = \frac{5 \times 16}{0.1} \Rightarrow v = 800\text{ms}^{-1}$$

Velocity of the bullet is 800ms^{-1}

Impulse: This is the product of force and time taken for the two colliding objects.

$$\text{Impulse} = F \times t.$$

Units are Ns

Or it is the change in momentum when two bodies collide.

Example:

1. A hammer hits a metal with a force of 200N . If the impact lasts for 0.4s , calculate the impulse due to this force.

$$\text{Impulse} = F \times t = 200 \times 0.4 = 80\text{Ns}$$

Third law: To every action there is an equal and opposite reaction.

